Deep Neural Ranking Models for Argument Retrieval

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Agenda

Introduction

Dataset and Models

Experiments and Results

Conclusion
Abstract

■ Task: Ranking arguments in a collection for the given query
■ Contributions
  • **RQ1.** How to shape useful training and validation set fit for the task of ad-hoc retrieval using the collection?
  • **RQ2.** Using neural ranking models that have shown good performance in ad-hoc retrieval tasks in the argument retrieval
    ▶ **RQ2.1.** Interaction-focused vs. representation-focused?
    ▶ **RQ2.2.** Static embedding vs. contextualized embedding?
    ▶ **RQ2.3.** Typical Neural ranking model vs. End-to-End?
  • **RQ3.** How to aggregate model results? Which strategy to use and what we require for doing so?
Outline

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  Ranking Task

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Why Argument Retrieval

- Different types of opinions toward controversial topics
- Getting an overview of every opinion is an exhaustive and time consuming task
- Automated decision making
- Opinion Summarization
What is Argument

- Argumentation unit which is composed of a claim (conclusion) and its premise [Rieke et al.(1997) Rieke, Sillars, and Peterson]
- Use the premises of one claim to support or attack other claims
- Claims could be a word, phrase or a sentence
- Premises are texts composed of multiple sentences or paragraphs
Argument components

Figure: The relation between the argument units ([Dumani(2019)])
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Ad-hoc Retrieval Task

- Heterogeneous Ranking Task
  - Typically queries are of a shorter length
  - Documents are longer texts

- Given the query, the task is to rank the existing documents in the collection

- Query Relevance Files: soft similarity scores for query-document pairs derived from the query log or click through data
  - qrel makes training the models possible

! We do not have the qrel file in our dataset
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Preprocessing and Visualisation
Query Relevance Information
Training and Validation sets
Deep Neural Ranking Models

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Args.me Corpus

387740 annotated arguments in total from crawling 4 debate portals (json format):

- Debatewise (14000 arguments)
- IDebate.org (13000 arguments)
- Debatepedia (21000 arguments)
- Debate.org (338000 arguments)

Information for each argument:

- unique ID
- claim
- premise
- source of crawling
- time of crawling
- stance of premise regard to claim
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Preprocessing and Visualisation: Claims

- Forming normalized claims
  - punctuation removal and case sensitivity
  - stop words removal

- Visualization and Statics
  - 66473 unique claims
  - 29970 unique tokens

Figure: Histogram of the unique claims based on the number of tokens.
Preprocessing and Visualisation: Claims

**Table:** Normalized claims with the highest number of premises

<table>
<thead>
<tr>
<th>norm cons</th>
<th>number of premises</th>
</tr>
</thead>
<tbody>
<tr>
<td>abortion</td>
<td>2401</td>
</tr>
<tr>
<td>gay marriage</td>
<td>1259</td>
</tr>
<tr>
<td>rap battle</td>
<td>1256</td>
</tr>
<tr>
<td>god exists</td>
<td>942</td>
</tr>
<tr>
<td>death penalty</td>
<td>941</td>
</tr>
</tbody>
</table>
Tokenizing punctuation
- for static embedding: god exists. ⇒ god exists <PERIOD>
- for contextualized embedding is not required!

Removing consecutive repetitive tokens
- !!!!!!!! ⇒ <EXCLAMATIONMARK>
- yes yes yes ⇒ yes

Mapping digits to words
- 95 ⇒ ninety-five

Removing the URLs
- http://example.net/achiever.html?boy=armyauthority=beginner
Preprocessing and Visualisation: Premises

- Statistics of the premises:
  - vocabulary size: 586796
  - 85% of the premises have the length of less than 200 words

- Arguments with the premise length of less than 15 tokens are removed

Figure: Histogram of the premises based on their length (number of tokens separated by white space)
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Learning to Rank

- Learning goal: related documents over the unrelated ones
- Pairwise hinge cost function
- Relevant and irrelevant Query-Document pairs are required and are missing in the corpus
- A model to produce the similarity scores (We use Deep ranking models)

**Figure:** Hinge as a pairwise cost function
RQ.1: Useful dataset for ad-hoc task

- Distant Supervision Approach
  - Claims ⇒ Queries
  - Premises ⇒ Related Documents

- Unrelated premise for each query
  - qrel files contain also unrelated query-document pairs
  - similarity measure: fuzzy similarity
  - premise of an unrelated claims could be an unrelated document to our claims

- A binary query relevance is formed ⇒ Exploitation of deep ranking models in the context of argument retrieval is possible now!
Dataset Ready for Ad-hoc Task

Data collection ready for the ad-hoc task (for static and contextualized embedding) with the following columns:

**Important Note:** Different arguments may have same claims and different premises

<table>
<thead>
<tr>
<th>id</th>
<th>claim</th>
<th>norm-claim</th>
<th>premise</th>
<th>unrelated id</th>
<th>unrelated premise</th>
</tr>
</thead>
<tbody>
<tr>
<td>arg₁</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>arg₂</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>
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Training and Validation Sets

- Training set: 312248 arguments with one unrelated document each
- Validation set: 4885 arguments: 20 unrelated documents each

Figure: Different datasets and their number of arguments
Validation Arguments

RQ.1: Forming an appropriate training and validation dataset

Figure: An ideal ranking for a validation query
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Neural Ranking Models

- **Applications**: ad-hoc retrieval, question answering, automatic conversation

- **Similarity of input pairs (query \( q \), document \( d \))**: 
  \[
  f(q, d) = g(\psi(q), \phi(d), \eta(q, d))
  \]

  - \( \psi(q) \), \( \phi(d) \) and \( \eta(q,d) \) are representation of the texts \( q \), \( d \) and the pair of \( q \) and \( d \) respectively

- **Representation-focused** and **Interaction-focused networks**
Exploited Models

<table>
<thead>
<tr>
<th>Model</th>
<th>type</th>
<th>embedding</th>
<th>re-rank</th>
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<tbody>
<tr>
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<td>rep</td>
<td>static</td>
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</tr>
<tr>
<td>DRMM</td>
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</tr>
<tr>
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<td>yes</td>
</tr>
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<td>yes</td>
</tr>
<tr>
<td>Vanilla BERT</td>
<td>int</td>
<td>contx</td>
<td>yes</td>
</tr>
<tr>
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<td>int</td>
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<tr>
<td>SNRM</td>
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## Siamese Network

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*Figure: Similarity scores using recurrent neural network*
DRMM: Deep Relevance Matching Model

- Interaction-focused network
- Matching histogram of the query and document token embedding as the input to a fully connected network for similarity score

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**KNRM: Kernel-based Neural Ranking Model**

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- Another strategy for encoding the input pair interaction
- Forming translation matrix: elements are the cosine similarity of the term embedding
- Applying the RBF as the kernels and forming the input features for fully connected network
- A linear layer learns the score similarity of the input pairs
CKNRM: Covolutional KNRM

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- Using **Convolutional** windows to get a representation of document and query n-grams
- Forming cross-match layer instead of translation matrix for encoding the interaction of the n-grams in document and query
- The idea of applying the RBF and linear layer for computing the similarity score remain the same!
Ranking Models with Contextualized Embedding

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<td>contx</td>
<td>yes</td>
</tr>
<tr>
<td>SNRM</td>
<td>rep</td>
<td>static</td>
<td>no</td>
</tr>
</tbody>
</table>

- BERT base uncased as the contextualized embedding
- Embedding dimension of the tokens: 768
- Ranking models used with BERT:
  - Vanilla-BERT: linear layer at the top of BERT network
  - BERT and DRMM
  - BERT and KNRM
SNRM: Stand alone Neural Ranking Model

All the models up to now require candidate documents to do a re-ranking: Their inference is a 2 step process (candidate selector is BM25 for our case)

- Propagation of the error from the first ranker mode (in our case BM25)
- SNRM as an end-to-end ranking model
  - Hour-glass shape networks for generating representation of the n-grams of the inputs
  - Constructing an inverted index of the documents
  - L1 regularization term in the cost function
Figure: Training process of SNRM ([Zamani et al.(2018)Zamani, Dehghani, Croft, Learned-Miller, and Kamps])
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Train and Validation Phase

- 10000 sample data for hyper-parameter tuning and debug the codes so that the models run correctly
- Query length: 20 and Document length: 100

Each batch: 32 argument

Train the models
  - static embedding: 10 epochs
  - contextualized embedding: 5 epochs

Validation run for 8 times within a training epoch
  - Top 20 hits among the 105 validation documents for each query
  - Validation metrics: MRR@20, MAP@20, and nDCG@20
  - For binary qrel: MAP@20 more stable validation scores
Sample Training and Validation Curves

(a) DRMM

(b) Vanilla BERT
Validation Results

- **RQ2.1:** Representation-focus vs. interaction-focus
- **RQ2.2:** Contextualized and Static Embedding
- **RQ2.3:** Typical Neural ranking model vs. End-to-End?

### Table: Models

<table>
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<tr>
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<tbody>
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<td>GRU</td>
<td>rep</td>
<td>static</td>
<td>yes</td>
<td>0.241</td>
</tr>
<tr>
<td>DRMM</td>
<td>int</td>
<td>static</td>
<td>yes</td>
<td>0.528</td>
</tr>
<tr>
<td>KNRM</td>
<td>int</td>
<td>static</td>
<td>yes</td>
<td>0.727</td>
</tr>
<tr>
<td>CKNRM</td>
<td>int</td>
<td>static</td>
<td>yes</td>
<td>0.733</td>
</tr>
<tr>
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<td>int</td>
<td>contx</td>
<td>yes</td>
<td>0.88</td>
</tr>
<tr>
<td>DRMM BERT</td>
<td>int</td>
<td>contx</td>
<td>yes</td>
<td>0.881</td>
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<tr>
<td>KNRM BERT</td>
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<td>contx</td>
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<td>0.902</td>
</tr>
<tr>
<td>SNRM</td>
<td>rep</td>
<td>static</td>
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</tr>
</tbody>
</table>
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Re-ranking Candidate Arguments

- 50 test queries provided in the Touché task
- 100 first hits by each model for each test query is saved

Figure: Candidate documents to be re-ranked in the test phase
Inference in SNRM

**Figure:** Document retrieval process
([Zamani et al. (2018)](Zamani, Dehghani, Croft, Learned-Miller, and Kamps))
RQ3. Aggregation Strategy

Why to aggregate?
- Performance improvement
- Aggregation of the different model principles

How to aggregate?
- Using regression between the normalized model scores

What do we need to know before the regression?
- How diverse the model results are.
- Models with outlier results. Assumption: Outlier results belong to weak models!
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Model Output Analysis

- The model results are vectors: retrieved documents as dimensions and scores are the values in each dimension. Retrieved documents are not the same for the models.
- Jaccard and Spearman Coefficients for measuring the similarity of the ranking results:
  - Jaccard: portion of the documents in common
  - Spearman: correlation of the ranking scores of the common documents
- The average of the coefficients over 50 test queries are calculated.
Jaccard Coefficient as Similarity Measure

Jaccard: portion of the documents in common $J(A, B) = \frac{|A \cap B|}{|A \cup B|}$

**Figure:** The heat map of the Jaccard coefficient for the 50 test queries
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Linear Regression as an Aggression Strategy

- We assume SNRM results as outlier data (Based on the similarity results)
- Regression model is trained on validation set (1 related and 1 unrelated document)
  - 2 * 4885 data points for training the regression with the dimension of 7
- Union of the retrieved documents by models are scored by the regression model
  - If a model did not retrieve a document, 0 is assigned to the corresponding dimension
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Argument Quality Dimensions

- **Logical**: acceptable and relevant premises to the arguments
- **Rhetorical**: the ability of convince the audiences
- **Dialectical** (utility): the ones by which a stance can be built
- Our concern in this study: Focusing on the **Logical** aspect
Test Results

- nDCG@5 score is calculated over the retrieved arguments
- Manually annotation is done by human annotators based on the different quality dimensions of the arguments

<table>
<thead>
<tr>
<th>Model</th>
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<th>embedding</th>
<th>re-rank</th>
<th>MAP@20</th>
<th>nDCG@5</th>
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<td>0.241</td>
<td>×</td>
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Test Results

- KNRM (our best performing model) ranked 4th in the competition
- Most of the competitors got less score than the baseline (Dirichlet LM)
  - Argument retrieval meeting the quality dimensions is not an easy task
- Validation results and test results were not correlated
  - related arguments \(\neq\) good arguments (meeting the argument quality dimensions)
  - Relevance is a required but not enough condition for a good argument
- Interaction-focused network outperformed representation-focused networks
  - Representation focused networks’ results are not shown in the table
- Aggregation model has been trained on the validation set and its MAP@20 score on the validation set is useless.
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Summary

Future Works
Summary

- **RQ1.** How to shape useful training and validation set fit for the task of ad-hoc retrieval from the collection?
  - ✓ Using distant super vision and assigning unrelated documents with Fuzzy similarity
  - ✓ Create validation set with higher number of unrelated documents

- Using neural ranking models that have shown good performance in ad-hoc retrieval tasks in the argument retrieval
  - • **RQ2.1.** Interaction-focused vs representation-focused
    - ✓ Representation-focused
  - • **RQ2.2.** Static embedding vs. contextualized embedding?
    - ✓ Contextualized embedding
  - • **RQ2.3.** Typical Neural ranking model vs. End-to-End?
    - ✓ Improvement needed for end-to-end approach

- **RQ3.** How to aggregate model results? Which strategy to use and what we require for doing so?
  - ✓ Linear regression as an aggregation strategy
  - ✓ Analysis of result similarity is required
Outline

Introduction

Dataset and Models

Experiments and Results

Conclusion
  Summary
  Future Works
What’s next...

- Providing a concrete mathematical definition of the argument quality dimensions to be included in the cost function of the networks
- Working on strategies to map the interaction of the input pairs
- Devising more intuitive structures to create sparse representation for end-to-end models
Thanks!
Evaluation Metrics: Mean Reciprocal Rank (MRR)

**Figure:** An example of MRR calculation

For each user:
- **User 1:** Relevant Item, Non-Relevant Item, Relevant Item
  - Reciprocal Rank: $1/3$
- **User 2:** Non-Relevant Item, Relevant Item, Non-Relevant Item
  - Reciprocal Rank: $1/2$
- **User 3:** Non-Relevant Item, Relevant Item, Non-Relevant Item
  - Reciprocal Rank: 1

**Mean Reciprocal Rank:**

$$\frac{1/3 + 1/2 + 1}{3} = 0.61$$
Evaluation Metrics: Mean Average Precision (MAP)

Figure: An example of MAP calculation
Evaluation Metrics: Normalized Discounted Cumulative Gain (nDCG)

\[
DCG_p = \sum_{i=1}^{p} \frac{rel_i}{\log_2(i + 1)}
\]

\[
nDCG_p = \frac{DCG_p}{IDCG_p}.
\]
