Chapter IR:III

III. Retrieval Models

- Overview of Retrieval Models
- Boolean Retrieval
- Vector Space Model
- Binary Independence Model
- Okapi BM25
- Divergence From Randomness
- Latent Semantic Indexing
- Explicit Semantic Analysis
- Language Models

- Combining Evidence
- Learning to Rank

© HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [Boolean] [VSM] [BIM] [BM25] [LSI] [ESA] [LM]

Document representations $\mathbf{D}$.

- $T = \{t_1, \ldots, t_m\}$ is the set of m index terms (lemmatized or stemmed words).
- T are the atoms of a logical formula for d with operators $\wedge$, $\vee$, $\neg$, and brackets.
- $\mathbf{d} = (\bigwedge_{t \in d} t) \wedge \neg(\bigwedge_{t \notin d} t)$, where $\mathcal{I}_{\mathbf{d}}(t) = 1$ if t occurs in d, and $\mathcal{I}_{\mathbf{d}}(t) = 0$ otherwise.

Query representations $\mathbf{Q}$.

- $\mathbf{q}$ is a logical formula over T.

Relevance function ρ.

- $\rho(d, q) = \mathcal{I}(\mathbf{d} \rightarrow \mathbf{q})$, where $\rightarrow$ is the logical implication.
- $\rho(d, q) = 1$ indicates relevance of d to q, and $\rho(d, q) = 0$ otherwise.
- $R_q \subseteq D$ is the set of documents $d \in D$ relevant to q, i.e., with $\rho(d, q) = 1$.
- $\rho'(d, q) = P(\mathcal{I}(\mathbf{d} \rightarrow \mathbf{q}) = 1) = P(\mathbf{d} \rightarrow \mathbf{q}) = P(q \mid d)$ relaxes relevance scoring.

© HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [Boolean] [VSM] [BIM] [BM25] [LSI] [ESA] [LM]

Document representations $\mathbf{D}$.

- $T = \{t_1, \ldots, t_m\}$ is the set of m index terms (lemmatized or stemmed words).
- T are the atoms of a logical formula for d with operators $\wedge$, $\vee$, $\neg$, and brackets.
- $\mathbf{d} = (\bigwedge_{t \in d} t) \wedge \neg(\bigwedge_{t \notin d} t)$, where $\mathcal{I}_\mathbf{d}(t) = 1$ if t occurs in d, and $\mathcal{I}_\mathbf{d}(t) = 0$ otherwise.

Query representations $\mathbf{Q}$.

- $\mathbf{q}$ is a logical formula over T.

Relevance function ρ.

- $\rho(d, q) = \mathcal{I}(\mathbf{d} \rightarrow \mathbf{q})$, where $\rightarrow$ is the logical implication.
- $\rho(d, q) = 1$ indicates relevance of d to q, and $\rho(d, q) = 0$ otherwise.
- $R_q \subseteq D$ is the set of documents $d \in D$ relevant to q, i.e., with $\rho(d, q) = 1$.
- $\rho'(d, q) = P(\mathcal{I}(\mathbf{d} \rightarrow \mathbf{q}) = 1) = P(\mathbf{d} \rightarrow \mathbf{q}) = P(q \mid d)$ relaxes relevance scoring.

© HAGEN/POTTHAST/STEIN 2023

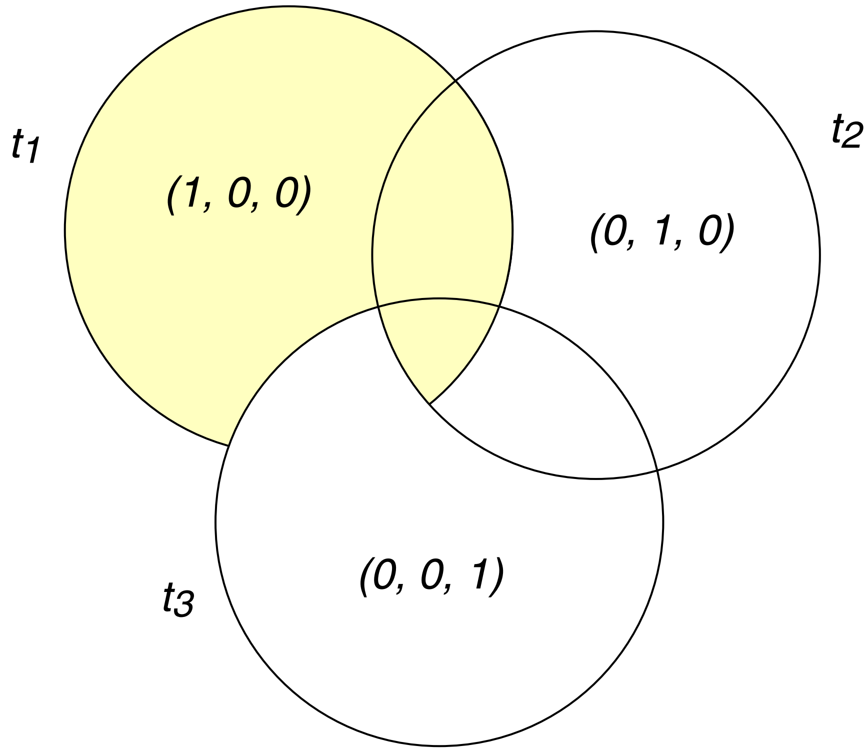
Remarks:

- $\mathcal{I} : T \rightarrow \{0, 1\}$ and $\mathcal{I} : \{\alpha \mid \alpha$ is a logical formula over $T\} \rightarrow \{0, 1\}$ is the evaluation or interpretation function that assigns truth values to the atoms T as well as to propositional formulas over them.

© HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Relevance Function ρ

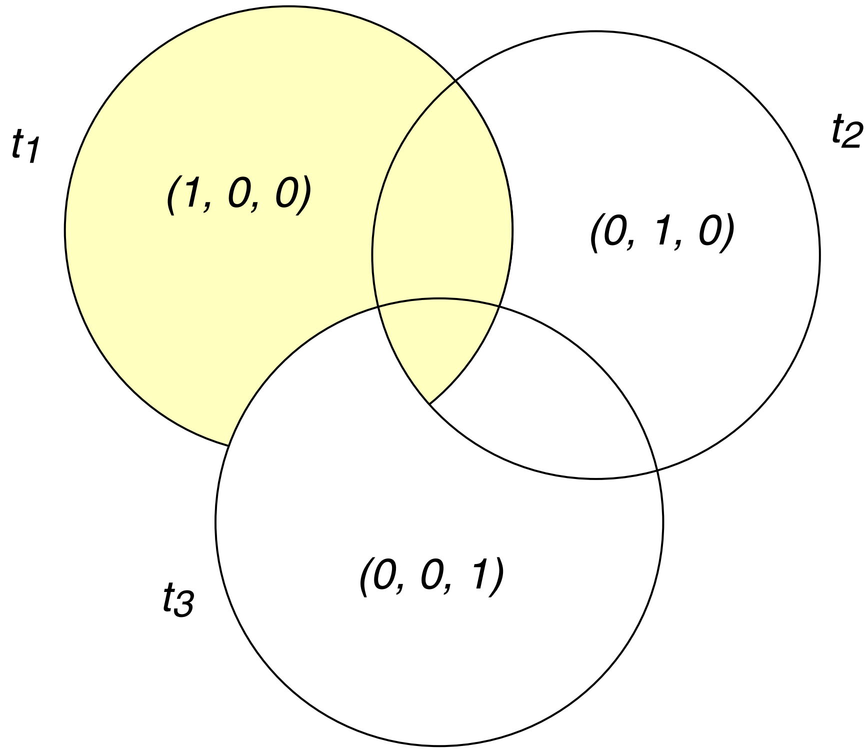


What query is illustrated?

© HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Relevance Function ρ



What query is illustrated?

$$\mathbf{q} \; = \; t_1 \wedge (t_2 \vee \neg t_3) \; \equiv \; (t_1 \wedge \neg t_2 \wedge \neg t_3) \vee (t_1 \wedge t_2 \wedge \neg t_3) \vee (t_1 \wedge t_2 \wedge t_3)$$

© HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval
Example

Document representation:

$$\mathbf{d} = \texttt{chrysler} \wedge \texttt{deal} \wedge \texttt{usa} \wedge \texttt{china} \wedge \neg\texttt{cat} \wedge \texttt{sales} \wedge \neg\texttt{dog} \wedge \ldots$$

Query representation:

$$\begin{aligned}\mathbf{q} &= \texttt{usa} \wedge (\texttt{dog} \vee \neg\texttt{cat}) \\ &\equiv (\texttt{usa} \wedge \texttt{dog}) \vee (\texttt{usa} \wedge \neg\texttt{cat}) \\ &\equiv (\texttt{usa} \wedge \neg\texttt{dog} \wedge \neg\texttt{cat}) \vee \\ &\quad\; (\texttt{usa} \wedge \texttt{dog} \wedge \neg\texttt{cat}) \vee \\ &\quad\; (\texttt{usa} \wedge \texttt{dog} \wedge \texttt{cat})\end{aligned}$$

Relevance function:

$\rho(d, q) = \mathcal{I}(\mathbf{d} \rightarrow \mathbf{q}) = 1,$ since $\mathcal{I}_{\mathbf{d}}(\texttt{usa}) = 1, \mathcal{I}_{\mathbf{d}}(\texttt{dog}) = 0,$ and $\mathcal{I}_{\mathbf{d}}(\texttt{cat}) = 0$.

© HAGEN/POTTHAST/STEIN 2023

Remarks:

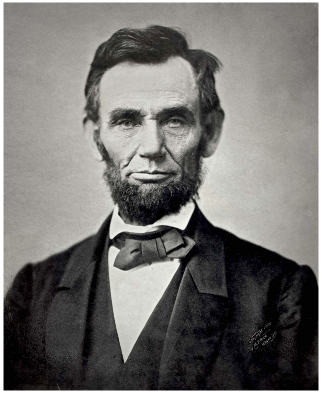
- The symbol "$\equiv$" denotes "is logically equivalent with".
- What does logical equivalence mean?
- A Boolean query in disjunctive normal form can be answered straightforward using an inverted index in parallel for each conjunction.
- A Boolean query in canonical disjunctive normal form will retrieve each document only once.

 ©HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Query Refinement: “Searching by Numbers”

Best practice in Boolean retrieval: (re)formulate queries until the number of documents retrieved is manageable. Example: pages about President Lincoln.



1. `lincoln`

   Results: many pages about cars, places, people

2. `president` $\wedge$ `lincoln`

   A result: “Ford Motor Company today announced that Darryl Hazel will succeed Brian Kelley as president of Lincoln Mercury.”

3. `president` $\wedge$ `lincoln` $\wedge$ $\neg$`automobile` $\wedge$ $\neg$`car`

   Not a result: “President Lincoln’s body departs Washington in a nine-car funeral train.”

4. `president` $\wedge$ `lincoln` $\wedge$ $\neg$`automobile` $\wedge$ `biography` $\wedge$ `life` $\wedge$ `birthplace` $\wedge$ `gettysburg`

   Results: $\emptyset$

5. `president` $\wedge$ `lincoln` $\wedge$ $\neg$`automobile` $\wedge$ (`biography` $\vee$ `life` $\vee$ `birthplace` $\vee$ `gettysburg`)

   A result: “President’s Day – Holiday activities – crafts, mazes, word searches, . . . ’The Life of Washington’ Read the entire book online! Abraham Lincoln Research Site”

©HAGEN/POTTHAST/STEIN 2023

Boolean Retrieval

Discussion

Advantages:

- Precision: in principle, any subset of documents from a collection can be designated by a Boolean query
- as in data retrieval, other fields are possible (e.g., date, document type, etc.)
- simple, efficient implementation

Disadvantages:

- retrieval effectiveness depends entirely on the user
- cumbersome query formulation (e.g., expertise required)
- no possibility to weight query terms
- no ranking; binary relevance scoring is too restrictive for most practical purposes (exceptions: medical retrieval, patent retrieval, eDiscovery (law))
- the size of the result set is difficult to be controlled

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [Boolean] [VSM] [BIM] [BM25] [LSI] [ESA] [LM]

Document representations $\mathbf{D}$.

- $T = \{t_1, \ldots, t_m\}$ is the set of m index terms (word stems, without stop words).
- T is interpreted as set of dimensions of an m-dimensional vector space.
- $\omega : \mathbf{D} \times T \to \mathbf{R}$ is a term weighting function, quantifying term importance.
- $\mathbf{d} = (w_1, \ldots, w_m)^T$, where $w_i = \omega(\mathbf{d}, t_i)$ is the term weight of the i-th term in T.

Query representations $\mathbf{Q}$.

- $\mathbf{q} = (w_1, \ldots, w_m)^T$, where $w_i = \omega(\mathbf{q}, t_i)$ is the term weight of the i-th term in T.

Relevance function ρ.

- Distance and similarity functions φ serve as relevance functions.
- $\rho(d, q) = \varphi(\mathbf{d}, \mathbf{q}) = \mathbf{d}^T\mathbf{q}$, the scalar product of vectors $\mathbf{d}$ and $\mathbf{q}$.
- Normalizing $\mathbf{d}$ and $\mathbf{q}$ calculates cosine similarity.

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [Boolean] [VSM] [BIM] [BM25] [LSI] [ESA] [LM]

Document representations $\mathbf{D}$.

- $T = \{t_1, \ldots, t_m\}$ is the set of m index terms (word stems, without stop words).
- T is interpreted as set of dimensions of an m-dimensional vector space.
- $\omega : \mathbf{D} \times T \rightarrow \mathbf{R}$ is a term weighting function, quantifying term importance.
- $\mathbf{d} = (w_1, \ldots, w_m)^T$, where $w_i = \omega(\mathbf{d}, t_i)$ is the term weight of the i-th term in T.

Query representations $\mathbf{Q}$.

- $\mathbf{q} = (w_1, \ldots, w_m)^T$, where $w_i = \omega(\mathbf{q}, t_i)$ is the term weight of the i-th term in T.

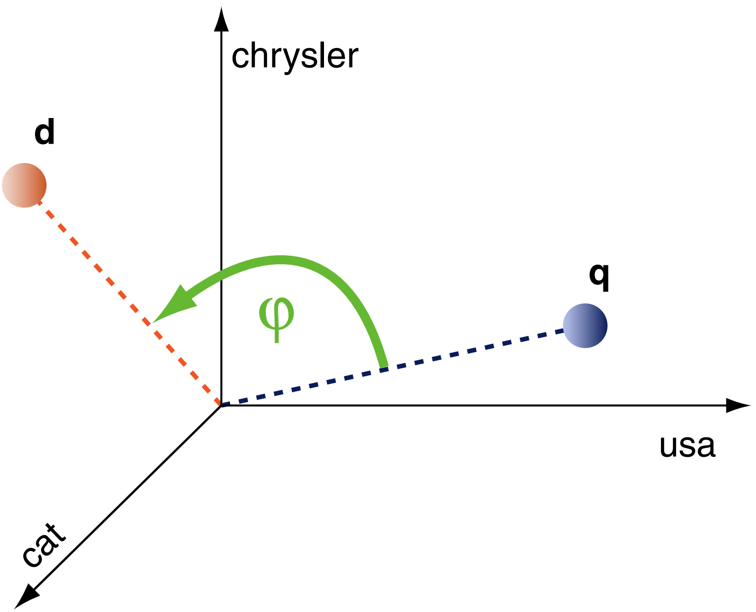
Relevance function ρ.

- Distance and similarity functions φ serve as relevance functions.
- $\rho(d, q) = \varphi(\mathbf{d}, \mathbf{q}) = \mathbf{d}^T\mathbf{q}$, the scalar product of vectors $\mathbf{d}$ and $\mathbf{q}$.
- Normalizing $\mathbf{d}$ and $\mathbf{q}$ calculates cosine similarity.

©HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Relevance Function ρ: Cosine Similarity



Vector Space Model

Relevance Function ρ: Cosine Similarity

The scalar product $\mathbf{a}^T\mathbf{b}$ between two m-dimensional vectors $\mathbf{a}$ and $\mathbf{b}$, where φ denotes the angle between them, is defined as follows:

$$\begin{aligned}\mathbf{a}^T\mathbf{b} &= ||\mathbf{a}|| \cdot ||\mathbf{b}|| \cdot \cos(\varphi) \\ \Leftrightarrow \cos(\varphi) &= \frac{\mathbf{a}^T\mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||},\end{aligned}$$

where $||\mathbf{x}||$ denotes the L2 norm of vector $\mathbf{x}$:

$$||\mathbf{x}|| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$$

Let $\rho(\mathbf{q}, \mathbf{d}) = \cos(\varphi)$ be the relevance function of the vector space model.

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Example

$$\mathbf{d} = \begin{pmatrix} \texttt{chrysler} & w_1 \\ \texttt{usa} & w_2 \\ \texttt{cat} & w_3 \\ \texttt{dog} & w_4 \\ \texttt{mouse} & w_5 \end{pmatrix} = \begin{pmatrix} \texttt{chrysler} & 1 \\ \texttt{usa} & 4 \\ \texttt{cat} & 3 \\ \texttt{dog} & 7 \\ \texttt{mouse} & 5 \end{pmatrix}$$

$$\mathbf{d}' = \begin{pmatrix} \texttt{chrysler} & 0.05 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.15 \\ \texttt{dog} & 0.35 \\ \texttt{mouse} & 0.25 \end{pmatrix}, \qquad \mathbf{q}' = \begin{pmatrix} \texttt{chrysler} & 0.2 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.2 \\ \texttt{dog} & 0.2 \\ \texttt{elephant} & 0.2 \end{pmatrix}$$

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Example

$$\mathbf{d} = \begin{pmatrix} \texttt{chrysler} & w_1 \\ \texttt{usa} & w_2 \\ \texttt{cat} & w_3 \\ \texttt{dog} & w_4 \\ \texttt{mouse} & w_5 \end{pmatrix} = \begin{pmatrix} \texttt{chrysler} & 1 \\ \texttt{usa} & 4 \\ \texttt{cat} & 3 \\ \texttt{dog} & 7 \\ \texttt{mouse} & 5 \end{pmatrix}$$

$$\mathbf{d}' = \begin{pmatrix} \texttt{chrysler} & 0.05 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.15 \\ \texttt{dog} & 0.35 \\ \texttt{mouse} & 0.25 \end{pmatrix}, \qquad \mathbf{q}' = \begin{pmatrix} \texttt{chrysler} & 0.2 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.2 \\ \texttt{dog} & 0.2 \\ \texttt{elephant} & 0.2 \end{pmatrix}$$

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Example

$$\mathbf{d} = \begin{pmatrix} \texttt{chrysler} & w_1 \\ \texttt{usa} & w_2 \\ \texttt{cat} & w_3 \\ \texttt{dog} & w_4 \\ \texttt{mouse} & w_5 \end{pmatrix} \quad = \quad \begin{pmatrix} \texttt{chrysler} & 1 \\ \texttt{usa} & 4 \\ \texttt{cat} & 3 \\ \texttt{dog} & 7 \\ \texttt{mouse} & 5 \end{pmatrix}$$

$$\mathbf{d}' = \begin{pmatrix} \texttt{chrysler} & 0.05 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.15 \\ \texttt{dog} & 0.35 \\ \texttt{mouse} & 0.25 \\ \texttt{elephant} & 0.0 \end{pmatrix}, \qquad \mathbf{q}' = \begin{pmatrix} \texttt{chrysler} & 0.2 \\ \texttt{usa} & 0.2 \\ \texttt{cat} & 0.2 \\ \texttt{dog} & 0.2 \\ \texttt{mouse} & 0.0 \\ \texttt{elephant} & 0.2 \end{pmatrix}$$

The angle φ between $\mathbf{d}'$ and $\mathbf{q}'$ is about $48°$, $\cos(\varphi) \approx 0.67$.

The weights in d' and q' denote the relative term frequency $w'_i = \frac{w_i}{\sum_{j=1}^{5} w_j}$. Dimensions are aligned with zero padding. The product ${\mathbf{d}'}^T \mathbf{q}' = 0.15$, the norms $||\mathbf{d}'|| = 0.5$ and $||\mathbf{q}'|| = 0.447$.

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Term Weighting: *tf* · *idf* [BIM Relevance Function]

To compute the weight w for a term t from document d under the vector space model, the most commonly employed term weighting scheme $\omega(t)$ is *tf* · *idf*:

- *tf* (t, d) denotes the normalized *term frequency* of term t in document d.
  The basic idea is that the importance of term t is proportional to its frequency in document d. However, t's importance does not increase linearly: the raw frequency must be normalized.

- *df* (t, D) denotes the *document frequency* of term t in document collection D.
  It counts the number of documents that contain t at least once.

- *idf* (t, D) denotes the *inverse document frequency*:

$$\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$$

  The importance of term t in general is inversely proportional to its document frequency.

A term weight ω for term t in document $d \in D$ is computed as follows:

$$\omega(t) = \mathit{tf}(t, d) \cdot \mathit{idf}(t, D).$$

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Term Weighting: *tf* · *idf* [BIM Relevance Function]

To compute the weight w for a term t from document d under the vector space model, the most commonly employed term weighting scheme $\omega(t)$ is *tf* · *idf*:

- *tf*(t, d) denotes the normalized *term frequency* of term t in document d.
  The basic idea is that the importance of term t is proportional to its frequency in document d. However, t's importance does not increase linearly: the raw frequency must be normalized.

- *df*(t, D) denotes the *document frequency* of term t in document collection D.
  It counts the number of documents that contain t at least once.

- *idf*(t, D) denotes the *inverse document frequency*:

$$\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$$

  The importance of term t in general is inversely proportional to its document frequency.

A term weight ω for term t in document $d \in D$ is computed as follows:

$$\omega(t) = \mathit{tf}(t, d) \cdot \mathit{idf}(t, D).$$

© HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Term Weighting: *tf* · *idf* [BIM Relevance Function]

To compute the weight w for a term t from document d under the vector space model, the most commonly employed term weighting scheme $\omega(t)$ is *tf* · *idf*:

- *tf*(t, d) denotes the normalized *term frequency* of term t in document d.
  The basic idea is that the importance of term t is proportional to its frequency in document d. However, t's importance does not increase linearly: the raw frequency must be normalized.

- *df*(t, D) denotes the *document frequency* of term t in document collection D.
  It counts the number of documents that contain t at least once.

- *idf*(t, D) denotes the *inverse document frequency*:

$$\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$$

  The importance of term t in general is inversely proportional to its document frequency.

A term weight ω for term t in document $d \in D$ is computed as follows:

$$\omega(t) = \mathit{tf}(t, d) \cdot \mathit{idf}(t, D).$$

©HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Term Weighting: *tf* · *idf* [BIM Relevance Function]

To compute the weight w for a term t from document d under the vector space model, the most commonly employed term weighting scheme $\omega(t)$ is *tf* · *idf*:

- *tf* (t, d) denotes the normalized *term frequency* of term t in document d.

  The basic idea is that the importance of term t is proportional to its frequency in document d. However, t's importance does not increase linearly: the raw frequency must be normalized.

- *df* (t, D) denotes the *document frequency* of term t in document collection D.

  It counts the number of documents that contain t at least once.

- *idf* (t, D) denotes the *inverse document frequency*:

$$\textit{idf}(t, D) = \log \frac{|D|}{\textit{df}(t, D)}$$

  The importance of term t in general is inversely proportional to its document frequency.

A term weight ω for term t in document $d \in D$ is computed as follows:

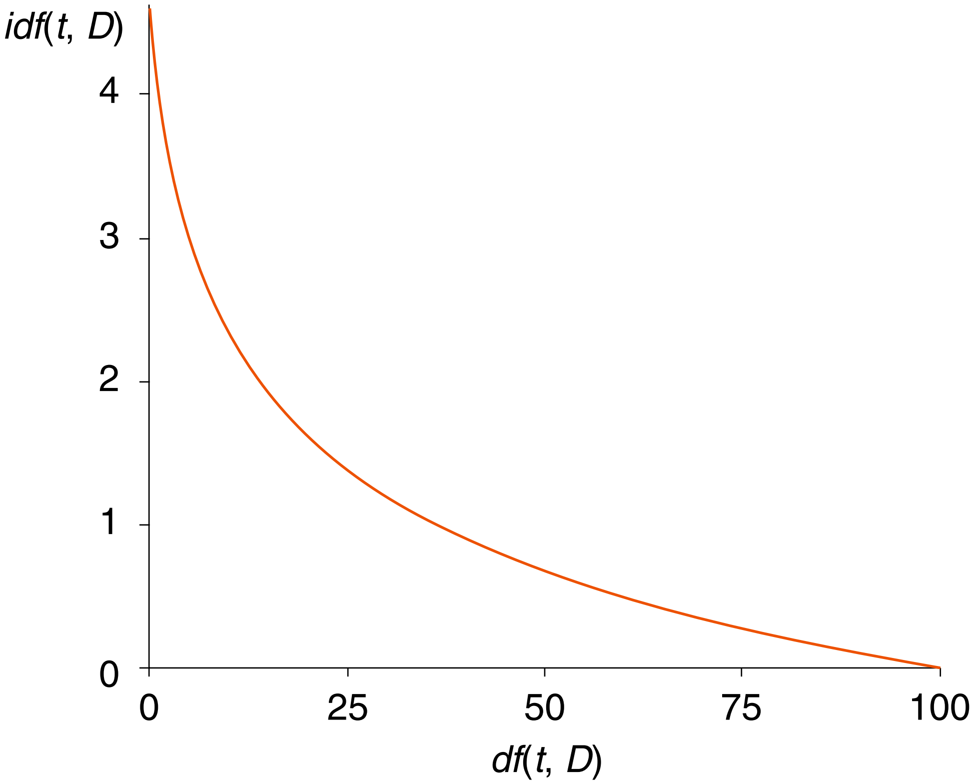
$$\omega(t) = \textit{tf}(t, d) \cdot \textit{idf}(t, D).$$

©HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Term Weighting: *tf* · *idf*

Plot of the function $\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$ for $|D| = 100$.



© HAGEN/POTTHAST/STEIN 2023

Remarks:

- Term frequency weighting was invented by Hans Peter Luhn: "There is also the probability that the more frequently a notion and combination of notions occur, the more importance the author attaches to them as reflecting the essence of his overall idea." [Luhn 1957]

- The importance of a term t for a document d is not linearly correlated with its frequency. Several normalization factors have been proposed [Wikipedia]:

  - $\mathit{tf}(t,d)/|d|$
  - $1+\log(\mathit{tf}(t,d))$ for $\mathit{tf}(t,d)>0$
  - $k+(1-k)\dfrac{\mathit{tf}(t,d)}{\max_{t'\in d}(\mathit{tf}(t',d))}$, where k serves as smoothing term; typically $k=0.4$

- Inverse document frequency weighting was invented by Karen Spärck Jones: "it seems we should treat matches on non-frequent terms as more valuable than ones on frequent terms, without disregarding the latter altogether. The natural solution is to correlate a term's matching value with its collection frequency." [Spärck Jones 1972]

- Spärck Jones gives little theoretical justification for her intuition. Given the success of *idf* in practice, over the decades, numerous attempts at a theoretical justification have been made. A comprehensive overview has been compiled by Robertson 2004.

- For example, interpreting the term $\frac{|D|}{\mathit{df}(t,D)}$ as inverse of the probability $P_{\mathit{df}}(t)=\frac{\mathit{df}(t,D)}{|D|}$ of t occurring in a random document in D yields $\mathit{idf}(t,D)=\log\frac{|D|}{\mathit{df}(t,D)}=-\log P_{\mathit{df}}(t)$. Logarithms fit relevance functions ρ since both are additive, yielding the interpretation: "The less likely (on a random basis) it is that a given combination of terms occurs, the more likely it is that a document containing this combination is relevant to the question." [Robertson 1972]

 ©HAGEN/POTTHAST/STEIN 2023

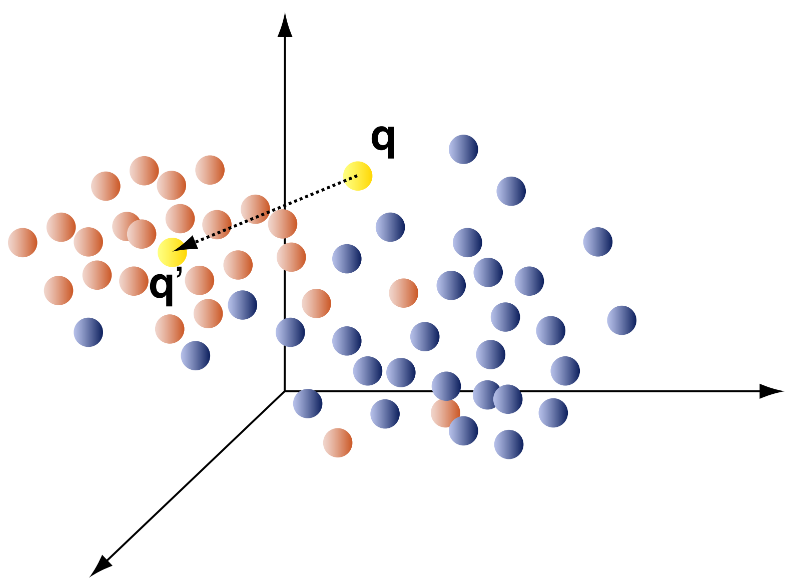
Vector Space Model

Query Refinement: Relevance Feedback

Given a result set R for a query q, and subsets $R^+ \subseteq R$ and $R^- \subseteq R$ of relevant and non-relevant documents, where $R^+ \cap R^- = \emptyset$, the query representation $\mathbf{q}$ can be refined with the document representations $\mathbf{R}$ using Rocchio's update formula:

$$\mathbf{q}' = \quad \alpha \cdot \mathbf{q} \quad + \quad \beta \cdot \frac{1}{|\mathbf{R}^+|} \sum_{\mathbf{d}^+ \in \mathbf{R}^+} \mathbf{d}^+ \quad - \quad \gamma \cdot \frac{1}{|\mathbf{R}^-|} \sum_{\mathbf{d}^- \in \mathbf{R}^-} \mathbf{d}^-,$$

where α, β, and γ adjust the impact of original query and (non-)relevant documents.



 © HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Query Refinement: Relevance Feedback

Given a result set R for a query q, and subsets $R^+ \subseteq R$ and $R^- \subseteq R$ of relevant and non-relevant documents, where $R^+ \cap R^- = \emptyset$, the query representation $\mathbf{q}$ can be refined with the document representations $\mathbf{R}$ using Rocchio's update formula:

$$\mathbf{q}' = \alpha \cdot \mathbf{q} + \beta \cdot \frac{1}{|\mathbf{R}^+|} \sum_{\mathbf{d}^+ \in \mathbf{R}^+} \mathbf{d}^+ - \gamma \cdot \frac{1}{|\mathbf{R}^-|} \sum_{\mathbf{d}^- \in \mathbf{R}^-} \mathbf{d}^-,$$

where α, β, and γ adjust the impact of original query and (non-)relevant documents.

Observations:

- Terms not in query q may get added; often a limit is imposed (say, 50).
- Terms may accrue negative weight; such weights are set to 0.
- Moves the query vector closer to the centroid of relevant documents.
- Works well if relevant documents cluster; less suited for multi-faceted topics.

Relevance feedback can be obtained directly from the user, indirectly through user interaction, or automatically assuming the top-retrieved documents as relevant.

©HAGEN/POTTHAST/STEIN 2023

Vector Space Model

Discussion

Advantages:

- Improved retrieval performance compared to Boolean retrieval
- Partial query matching: not all query terms need to be present in a document for it to be retrieved
- The relevance function ρ defines a ranking among the retrieved documents with respect to their computed similarity to the query

Disadvantages:

- Index terms are assumed to occur independent of one another

© HAGEN/POTTHAST/STEIN 2023